Condensation and superfluidity of exciton-polaritons in semiconductor microcavities

Y. G. Rubo

Centro de Investigación en Energía,
Universidad Nacional Autónoma de México, Mexico

Outline:

- Introduction: Additional light waves and polaritons
- Exciton-polaritons in semiconductor microcavities
- Observations of exciton-polariton condensation
- Half-vortices
- Half-vortices in the presence of TE-TM splitting
- Peculiarities of Kosterlitz-Thouless transition

Refraction of light. Monocromatic light

∼ 1670: Sir Isaac Newton demonstrated that a prism could decompose white light into a spectrum of colours.

I. Newton thought that the light consists of small particles— The corpuscular theory.

T. Young observed the phenomenon of interference that initiated the wave theory of light in XIX.

Different color means different frequency of electromagnetic radiation $\omega_{\text{red}} < \omega_{\text{blue}}$. The light with a fixed is referred as monocromatic light. Frequency $\omega = 2\pi c/\lambda$:

$$\lambda_{\text{red}} \simeq 700\,\text{nm}, \qquad \lambda_{\text{blue}} \simeq 400\,\text{nm}$$

Additional light waves and polaritons

XIX: Fresnel equations, Snell's cone, Brewster angle

1861-1862: Maxwell's equations. Constants $\epsilon(\omega)$ and $\mu(\omega)$. The refractive index $n(\omega) = \sqrt{\epsilon(\omega)\mu(\omega)}$.

The phase velocity of light the medium $v = c/n(\omega)$.
The wave vector $k = \omega/v = \omega n(\omega)/c$.

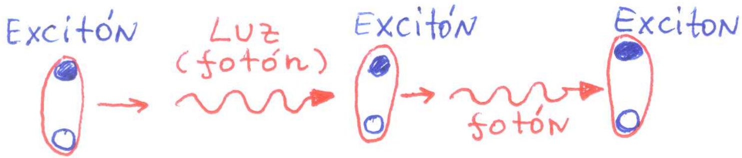
1950's: Effects of spacial dispersion, $\epsilon = \epsilon(\omega, k)$.

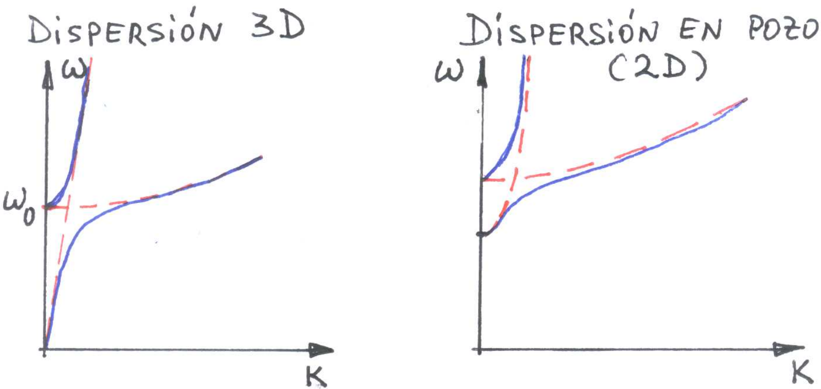
1957: S. I. Pekar discovered additional light wave in vicinity of exciton resonance.

1958: V.M.Agranovich, V.L.Ginsburg, D.G. Thomas, J.J. Hopfield. Tensor $\epsilon_{ij}(\omega, k)$ and the concept of exciton-polaritons.

Excitones y luz: polaritones

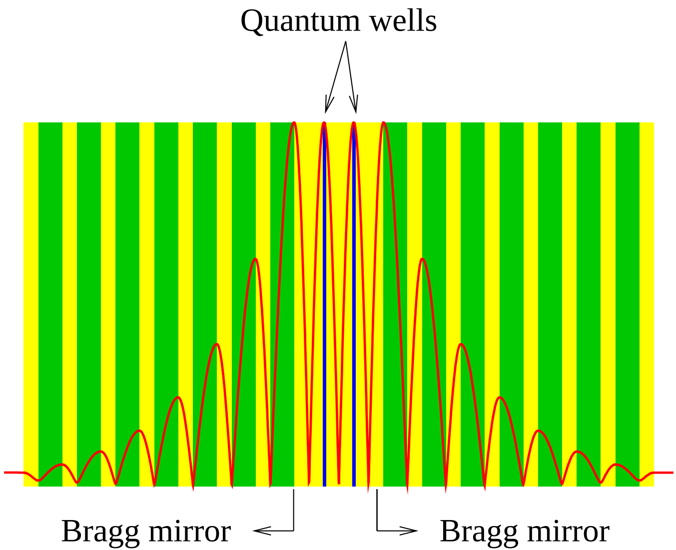
Interacin entre excitones y fotones produce una mezcla, polaritones.
(S.I.Pekar, V.M.Agranovich, V.L.Ginsburg, D.G. Thomas, J.J. Hopfield)



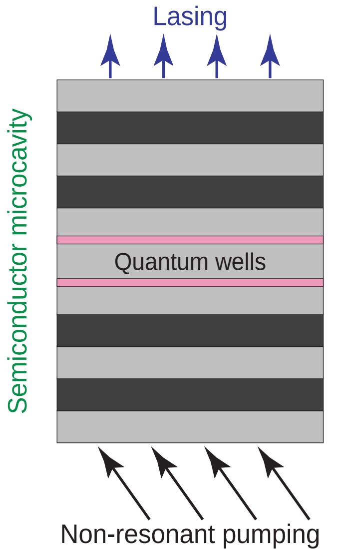


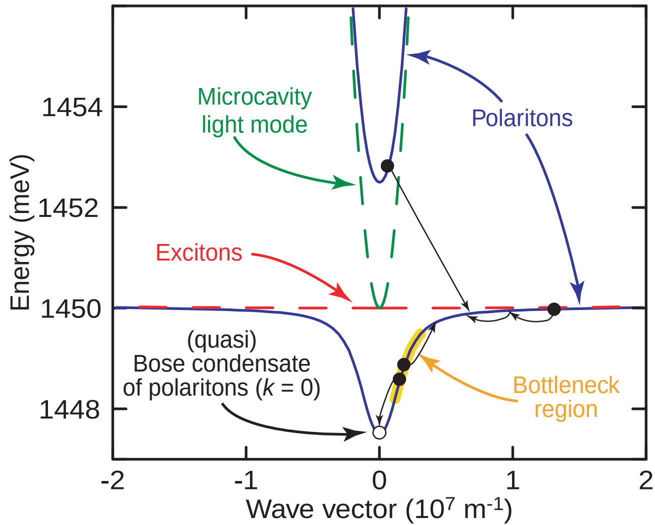
Semiconductor microcavities

To enhance the coupling between the light and electronic excitation: cavity light mode and quantum wells in the maximums of intensity.



Polariton condensation and lasing

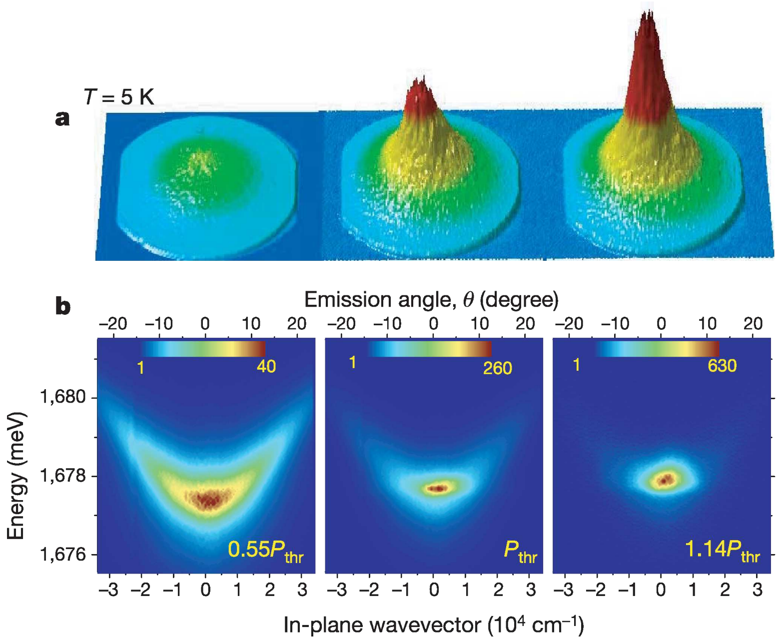




- Effective mass of lower branch polaritons: $m^* \sim 10^{-4} m_0$.
- Relaxation problem: bottleneck for $d\omega/dk > v_{\mathrm{sound}}$.
- Presence of polarization degree of freedom.
  The order parameter is complex 2D vector $\vec{\psi} \propto \vec{\mathcal{E}}_{\parallel}$.

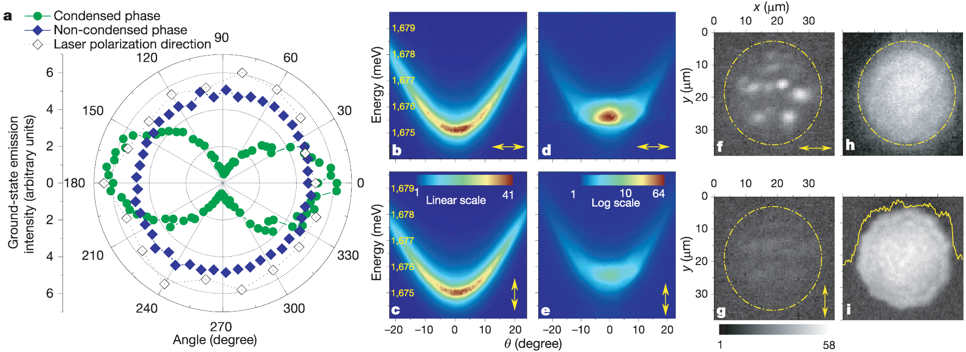
First observation of condensation (Le Si Dang group)

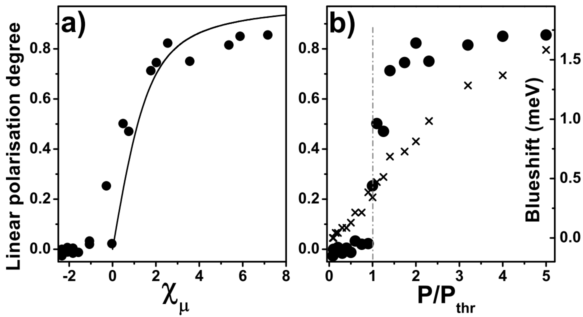
Emission from the CdTe-based microcavity:



From J. Kasprzak *et al.*, Nature 443, 409 (2006).

Spontaneous polarization formation (Le Si Dang group)





From J. Kasprzak *et al.*, Nature 443, 409 (2006) and Phys. Rev. B 75, 045326 (2007).

Linear polarization of the condensate

Formation of linear polarization in polariton condensates [Le Si Dang *et al.*; Snoke *et al.*, 2006] arises due to the reduction of polariton-polariton repulsion energy $H_{\rm int}$:

$$H_{\rm int} = \frac{1}{2}\int d^2r \left\{ (U_0 - U_1)(\vec{\psi}^* \cdot \vec{\psi})^2 + U_1|\vec{\psi}^* \times \vec{\psi}|^2 \right\}.$$

Two interaction constants, $U_0 = AM_{\uparrow\uparrow}$ and $U_1 = A(M_{\uparrow\uparrow} - M_{\uparrow\downarrow})/2$, where $A = \pi R^2$ is the excitation spot area. Typically, $U_0/2 < U_1 < U_0$.

At a fixed concentration $n = (\vec{\psi}^* \cdot \vec{\psi})$ minimum of $H_{\rm int}$ is reached for

$$\vec{\psi}^* \times \vec{\psi} = 0 \;\Rightarrow\; \text{Linear polarization}$$

One can write

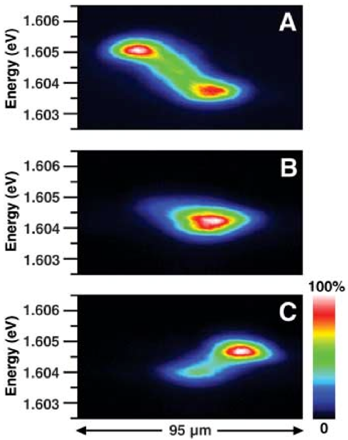
$$\vec{\psi}_{\rm lin} = \{\psi_x, \psi_y\} = \sqrt{n}\, e^{i\theta}\{\cos\eta, \sin\eta\},$$

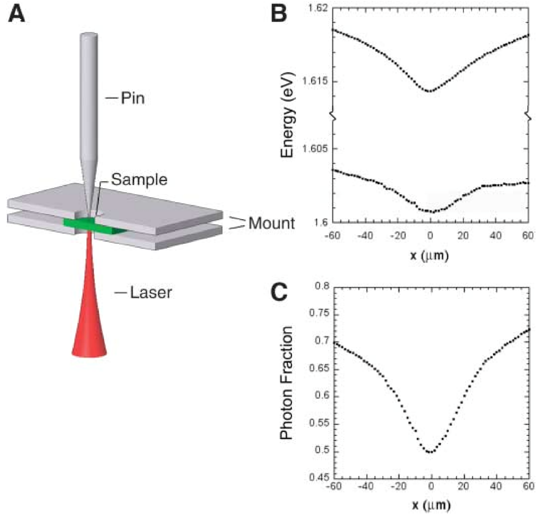
so that the order parameter is defined by two angles, η and θ.

Note that the states η, θ and $\eta + \pi, \theta + \pi$ are identical.

Condensation in the trap (Snoke group)

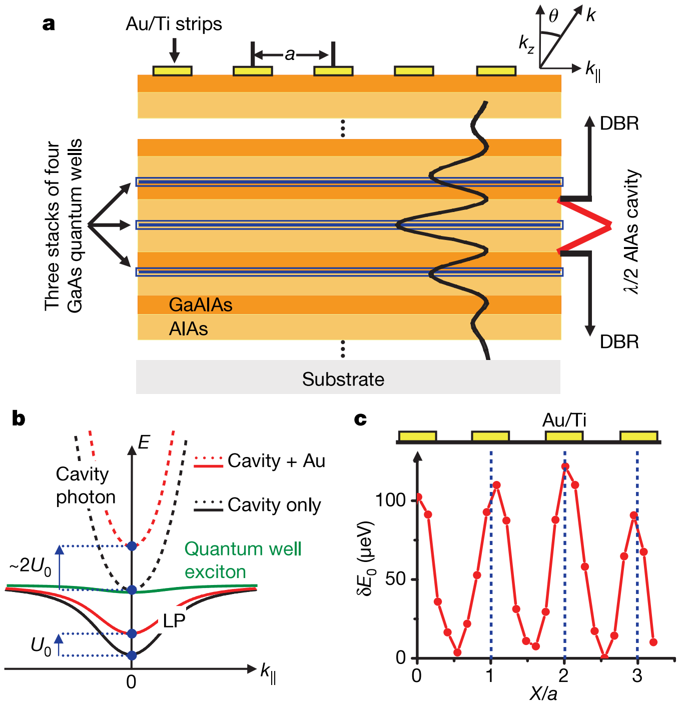
GaAs-based microcavity structure under non-uniform strain.





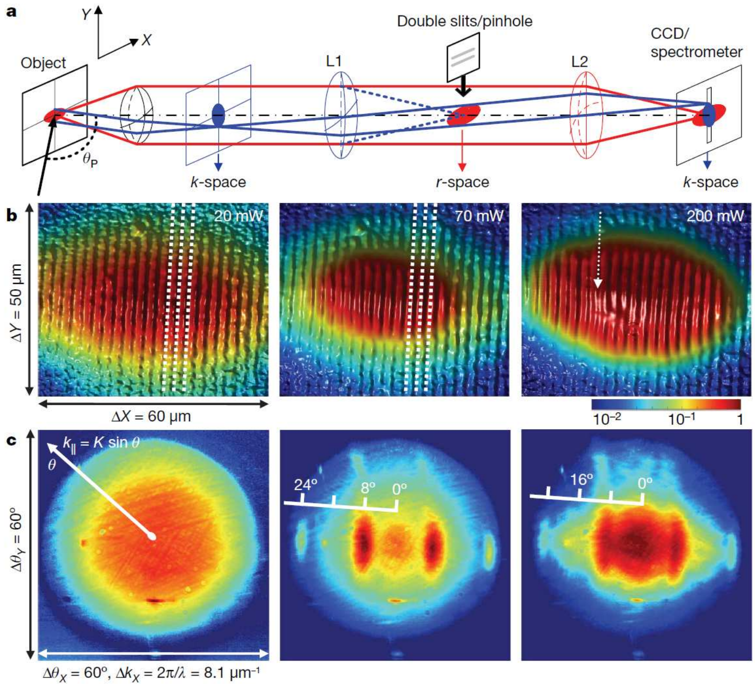
From R. Balili *et al.*, Science <u>316</u>, 1007 (2007).

Condensation in 1D superlattice (Yamamoto group)



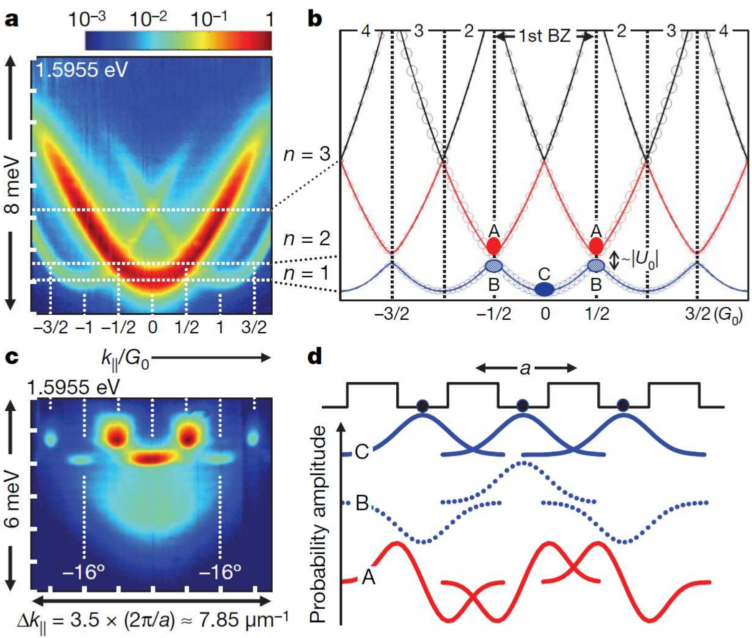
From C. W. Lai *et al.*, Nature 450, 529 (2007).

Condensation in 1D superlattice (Yamamoto group)



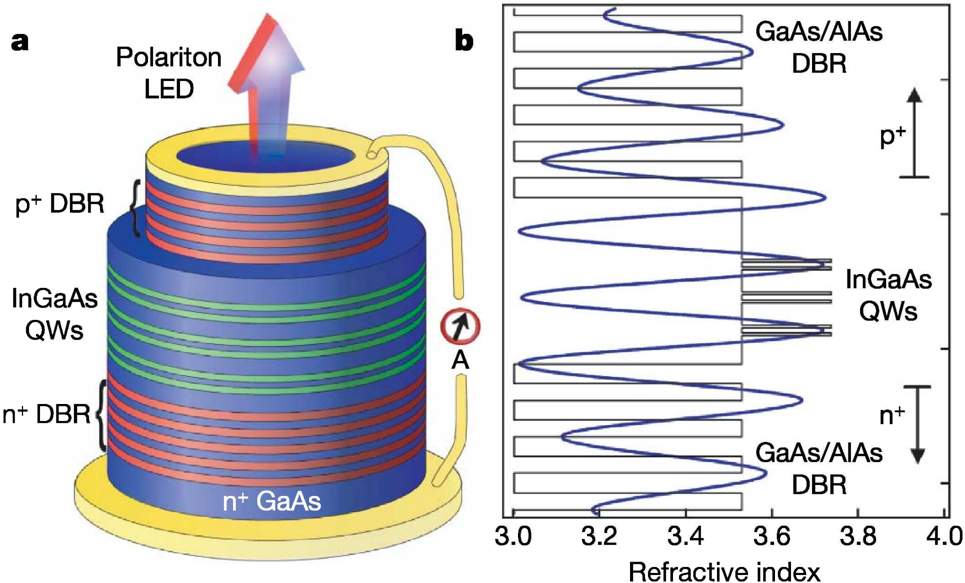
From C. W. Lai *et al.*, Nature <u>450</u>, 529 (2007).

Condensation in 1D superlattice (Yamamoto group)



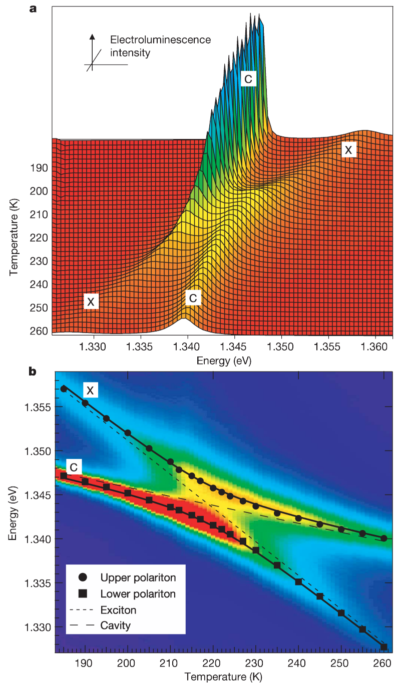
From C. W. Lai *et al.*, Nature 450, 529 (2007).

Electrically pumped polariton emission (Savvidis group)



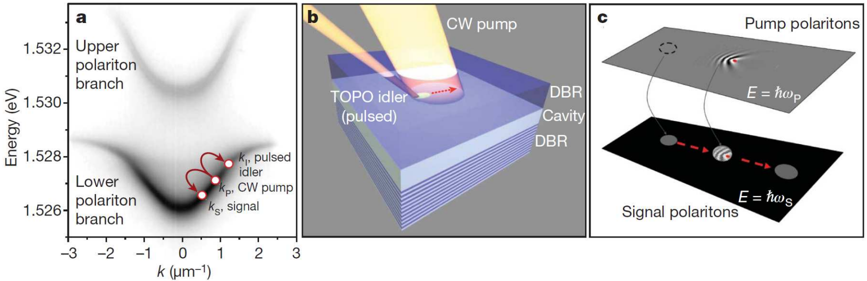
From S. I. Tsintzos *et al.*, Nature 453, 372 (2008).

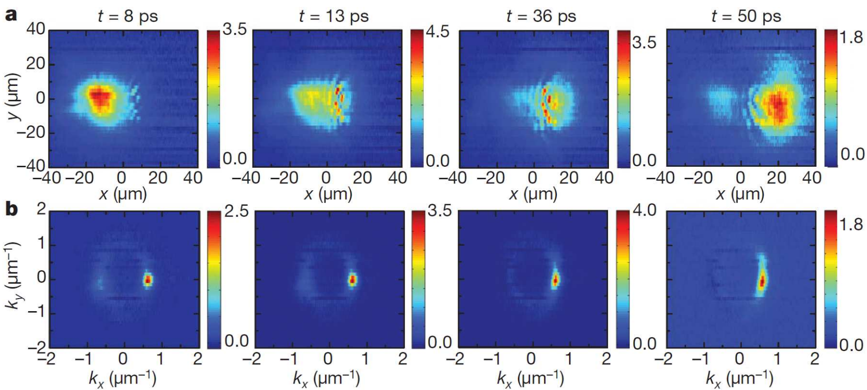
Electrically pumped polariton emission (Savvidis group)



From S. I. Tsintzos *et al.*, Nature 453, 372 (2008).

Observation of superfluid/soliton motion (Viña group)





From A. Amo *et al.*, Nature 457, 291 (2009).

The equilibrium Gross-Pitaevskii equation

The order parameter $\vec{\psi}$ is normalized by $(\vec{\psi}^{*} \cdot \vec{\psi}) = n$
The energy functional for the free-propagating polariton superfluid ($\hbar = 1$):

$$\mathcal{H} = H - \mu n = H_{\rm kin} + H_{\rm pot} + H_{\rm int} =$$
$$= \int d^2 r \left\{ \vec{\psi}^{*} \cdot \left(-\frac{1}{2m^*} \Delta - \mu \right) \vec{\psi} + V(\vec{r})(\vec{\psi}^{*} \cdot \vec{\psi}) + \frac{1}{2} \left[U_0 (\vec{\psi}^{*} \cdot \vec{\psi})^2 - U_1 \vec{\psi}^{*2} \vec{\psi}^{2} \right] \right\},$$

where $V(\vec{r})$ is regular or random potential.

The Gross-Pitaevskii equation then reads

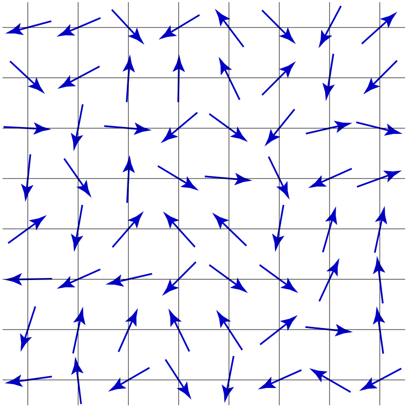
$$\mathrm{i} \frac{\partial \psi_i}{\partial t} = \frac{\delta \mathcal{H}}{\delta \psi_i^*} = \left[-\frac{1}{2m^*} \Delta - \mu \right] \psi_i + V(\vec{r}) \psi_i + U_0 \psi_j^* \psi_j \psi_i - U_1 \psi_j \psi_j \psi_i^* .$$

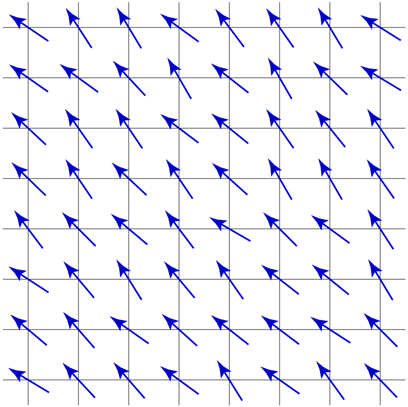
One can search for particular solutions, study the statistics from the Gibbs distribution $\propto \exp\{-\mathcal{H}/T^*\}$, and phase transitions.

Phase-transition in XY-model

The symmetry is lowered due to the disorder-order transition:

$T > T_c$ $T < T_c$



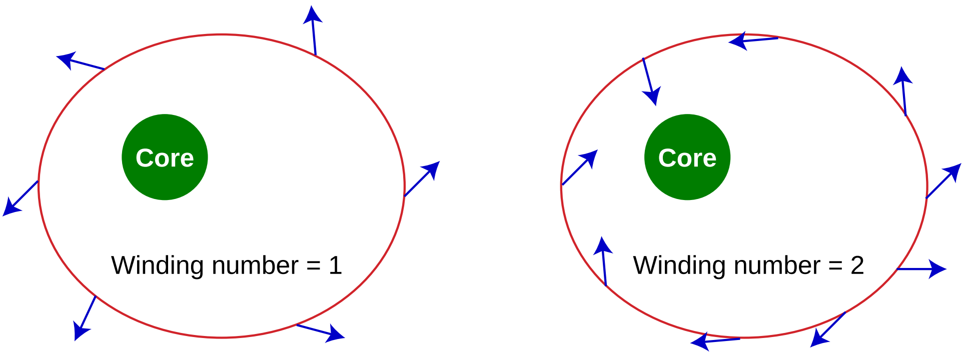


$$H = \rho_s \sum_{\langle ij \rangle} [1 - \cos(\theta_i - \theta_j)] = \frac{1}{2}\rho_s \sum_{\langle ij \rangle} [\theta(\vec{r}_i) - \theta(\vec{r}_j)]^2$$

$$= \frac{1}{2}\rho_s \sum_i a^2 |\nabla\theta(\vec{r}_i)|^2 = \frac{1}{2}\rho_s \int d^2r |\nabla\theta(\vec{r})|^2.$$

XY-model: Vortices and winding numbers

Apart from spin waves, the vortices are important (V. L. Berezinskii, 1970). In vortex, there is rotation of the spin on distances $\gg a$ (the core size), with a resulting change multiple by 2π:



Variation of $\theta(\vec{r})$ is subject to minimization of the Hamiltonian energy,

$$-\rho_s \Delta\theta(\vec{r}) = 0, \qquad \theta = \theta_0 + n_w \phi.$$

The winding number $n_w = 0, \pm 1, \pm 2, \ldots$. Mathematically, $\pi_1(S_1) = Z$.

The Berezinskii-Kosterlitz-Thouless (BKT) transition

The single vortex energy

$$E_s = \frac{1}{2}\rho_s \int d^2r |\nabla\theta|^2 = \frac{1}{2}\rho_s \int d^2r \left(\frac{1}{r}\frac{d\theta}{d\phi}\right)^2 = \pi\rho_s k^2 \int_a^R \frac{1}{r}dr = \pi\rho_s n_w^2 \ln\left(\frac{R}{a}\right).$$

The energy of a pair of vortices is finite for $n_1 + n_2 = 0$,

$$E_p = \pi\rho_s(n_1+n_2)^2 \ln(R/a) - 2\pi\rho_s n_1 n_2 \ln(r/a).$$

The critical temperature can be found by
[J. M. Kosterlitz and D. J. Thouless, (1973); J. M. Kosterlitz, (1974)]

$$F = E_s - TS = \pi\rho_s \ln(R/a) - T\ln(R^2/a^2) = (\pi\rho_s - 2T)\ln(R/a),$$

so that single vortices appear and destroy the order at

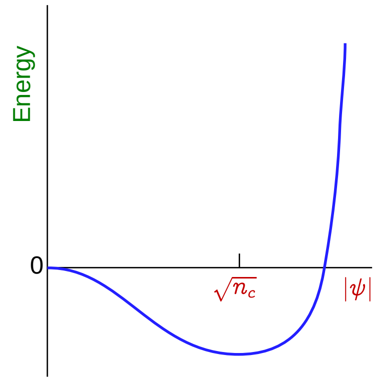
$$T_c = \frac{\pi}{2}\rho_s.$$

Mean-field theory for interacting bosons

$$H = \int d^d r \left\{ \frac{\hbar^2}{2m^*} |\vec{\nabla}\psi(r)|^2 - \mu|\psi|^2 + \frac{1}{2}U_0|\psi|^4 \right\}.$$

$$T \to T_c : \ \mu \propto (T_c - T).$$

$$T \to 0 : \ \mu = U_0 n, \quad \psi = \sqrt{n_c}\, e^{i\theta}$$



Considering vortices one can assume $|\psi|^2 = n_c$ (otherwise *energy* $\propto$ *area*), so $\psi = \sqrt{n_c}\, e^{i\theta(\vec{r})}$, and

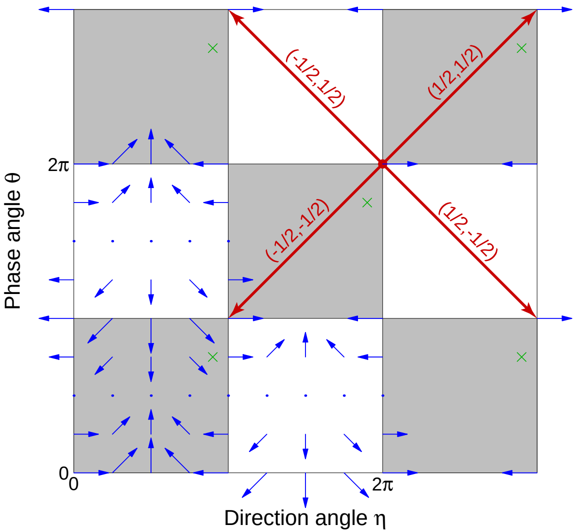
$$H_{el} = \frac{1}{2}\rho_s \int d^d r\, |\nabla\theta(\vec{r})|^2, \quad \rho_s = \frac{\hbar^2 n_c}{m^*}.$$

The vortex core. Condition $|\psi| = \sqrt{n_c}$ is violated on distances $r \lesssim a = \hbar/\sqrt{2m^*\mu}$, that define the vortex core.

Singularity at $r \to 0$, where $\psi \propto r e^{\pm i\phi},\ r^2 e^{\pm 2i\phi}, \ldots$ for $n_w = \pm 1,\ \pm 2, \ldots$.

The order parameter space

$$\vec{\psi}_{\rm lin} = e^{i\theta}\left\{\cos\eta, \sin\eta\right\}.$$



The possible changes are:
$\eta \to \eta + 2\pi k$,
$\theta \to \theta + 2\pi m$.

Vortex carries two topological charges (winding numbers), (k, m).

Integer vortices:
$k, m = 0, \pm1, \pm2, \ldots$

Half-integer vortices:
$k, m = \pm1/2, \pm3/2, \ldots$

Half-vortices

Half-vortices in ^{3}He-A: G.E. Volovik and V.P. Mineev, (1976);
M.C. Cross and W.F. Brinkman, (1977).

They appear due to combined spin-gauge symmetry:

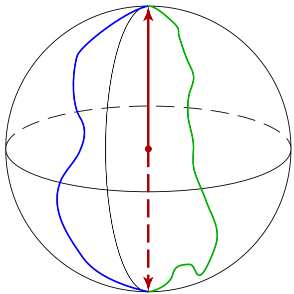
Spin quantization axis change $\vec{d} \to -\vec{d}$

Phase change $\theta \to \theta \pm \pi$

The superfluid velocity around the half-vortex $\vec{v}_s \propto \nabla\theta$ is a half of the superfluid velocity around the usual vortex with $\theta \to \theta \pm 2\pi$.

Half-vortex carries half-quantum of the superfluid current.

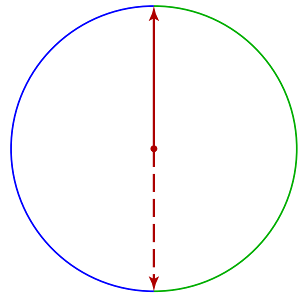
Why two winding numbers (k, m)?



Atomic spinor $s = 1$ condensates
(three-component)

3D real $\vec{d}$ and phase θ
Half-vortex: $\vec{d} \to -\vec{d}$, $\theta \to \theta + \pi$

All rotations $\vec{d} \to -\vec{d}$ are in the same homotopy class



Polariton pseudospin case
(two-component)

2D real $\vec{d}$ and phase θ
Half-vortex: $\vec{d} \to -\vec{d}$, $\theta \to \theta + \pi$

Clockwise and counterclockwise $\vec{d} \to -\vec{d}$ are topologically different

The half-vortex core

The core size $a = \hbar/\sqrt{2m^*\mu} \sim 1\ \mu\mathrm{m}$. For a basic half-vortex

$$\vec{\psi}_{\mathrm{hv}} = \sqrt{n}\left[\vec{A}(\phi)f(r/a) - \mathrm{i}\vec{B}(\phi)g(r/a)\right],$$

where the azimuthal dependencies are given by

$$\vec{A}(\phi) = \mathrm{e}^{im\phi}\{\cos(k\phi), \sin(k\phi)\},$$

$$\vec{B}(\phi) = \mathrm{sgn}(km)\mathrm{e}^{im\phi}\{\sin(k\phi), -\cos(k\phi)\},$$

and radial functions $f(r/a)$ and $g(r/a)$ are found from $\delta H/\delta\vec{\psi}^* = 0$:

$$f'' + \frac{1}{\xi}f' - \frac{1}{2\xi^2}(f-g) + \frac{1}{2}(\gamma-1)\omega g + [1 - f^2 - \gamma g^2]f = 0,$$

$$g'' + \frac{1}{\xi}g' - \frac{1}{2\xi^2}(g-f) + \frac{1}{2}(\gamma-1)\omega f + [1 - g^2 - \gamma f^2]g = 0,$$

where $\xi = r/a$, $\gamma = (U_0 + U_1)/(U_0 - U_1)$, and $\omega = \mathrm{sgn}(km)\Omega/nU_1$.
Conditions $f(0) = g(0)$, $f^2(\infty) + g^2(\infty) = 1$, $2f(\infty)g(\infty) = \omega$.

Half-vortex in circular polarizations

$$\vec{\psi}_{\rm elp} = \sqrt{n}e^{{\rm i}m\phi}\left\{f\cos(k\phi) - {\rm i}g\sin(k\phi), f\sin(k\phi) + {\rm i}g\cos(k\phi)\right\} =$$
$$= \sqrt{n/2}\left([f + {\rm sgn}(km)g]e^{{\rm i}(m-k)\phi}\left|\uparrow\right\rangle + [f - {\rm sgn}(km)g]e^{{\rm i}(m+k)\phi}\left|\downarrow\right\rangle\right),$$

where

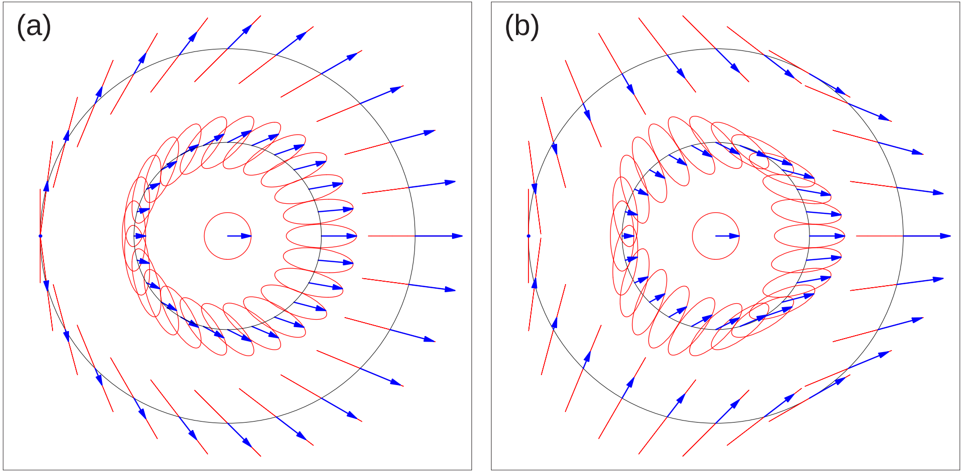
$$\left|\uparrow\right\rangle = \frac{1}{\sqrt{2}}\{1, {\rm i}\}, \qquad \left|\downarrow\right\rangle = \frac{1}{\sqrt{2}}\{1, -{\rm i}\},$$

Right half-vortices: $km > 0$. Left-circular component becomes fully depleted and polarization is right-circular at $r = 0$.

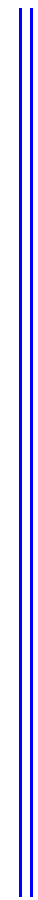
Left half-vortices: $km < 0$. Right-circular component becomes fully depleted and polarization is left-circular at $r = 0$.

The polarization texture of half-vortex core

Showing $\mathrm{Re}\{\vec{\psi}e^{-\mathrm{i}\omega t}\}$, where $\omega = \omega_p + \mu$.



Two left half-vortices



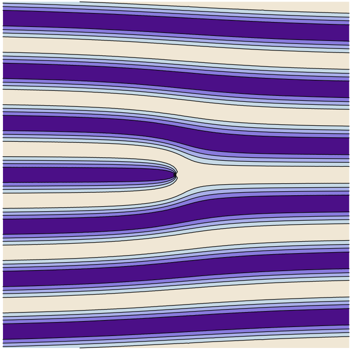
Pair of left half-vortices

Permuted pair of left half-vortices

Observation of half-vortices

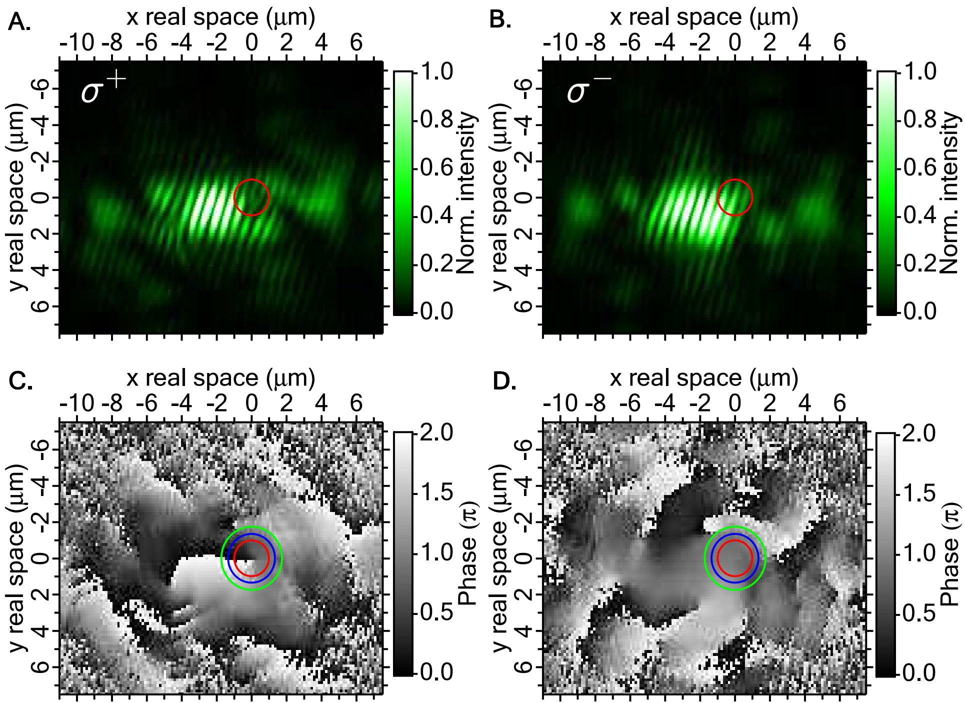
For one-component condensate one observes the interference pattern of two beams emitted by the same condensate.
One with vortex and the other without but inclined (plane wave):

$$|f(r)e^{i\phi} + e^{i\kappa y}|^2$$

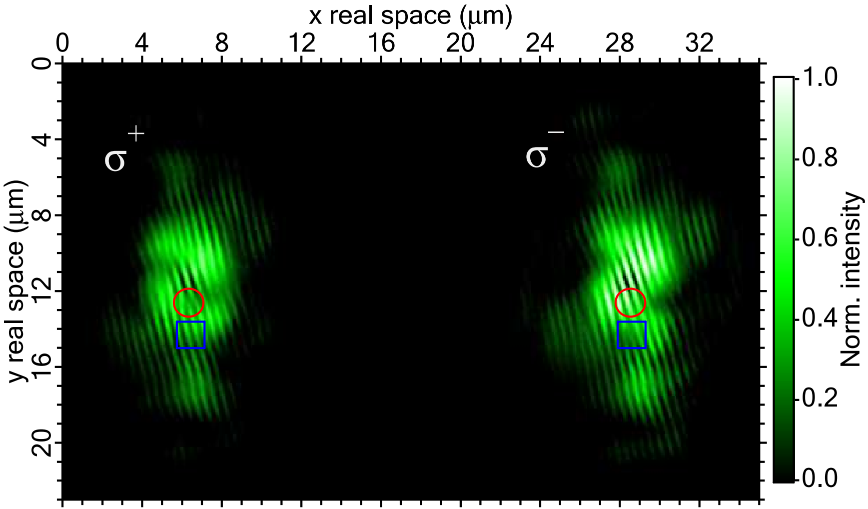


For polarized condensate one studies the interference patterns in both circular polarizations.
HQV: fork in one circular polarization and regular fringes in the other.

Experiment by Lagoudakis et al. (1)



Experiment by Lagoudakis et al. (2)



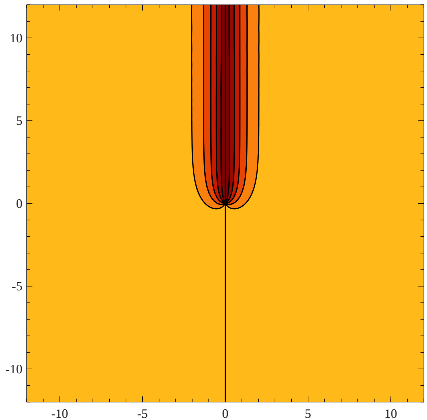
Close pair of $(-1/2, +1/2)$ (in red circle) and $(+1/2, +1/2)$ (in blue box).
These HQV form pure phase vortex $(0, +1)$ when placed together.
Their close position is an indication of polarization pinning.

Polarization pinning. Half-vortices with strings

$$E_{\text{el}} = \frac{1}{2}\rho_s \int d^2r \left\{(\nabla\theta)^2 + (\nabla\eta)^2 + \epsilon[1-\cos(2\eta)]\right\}$$

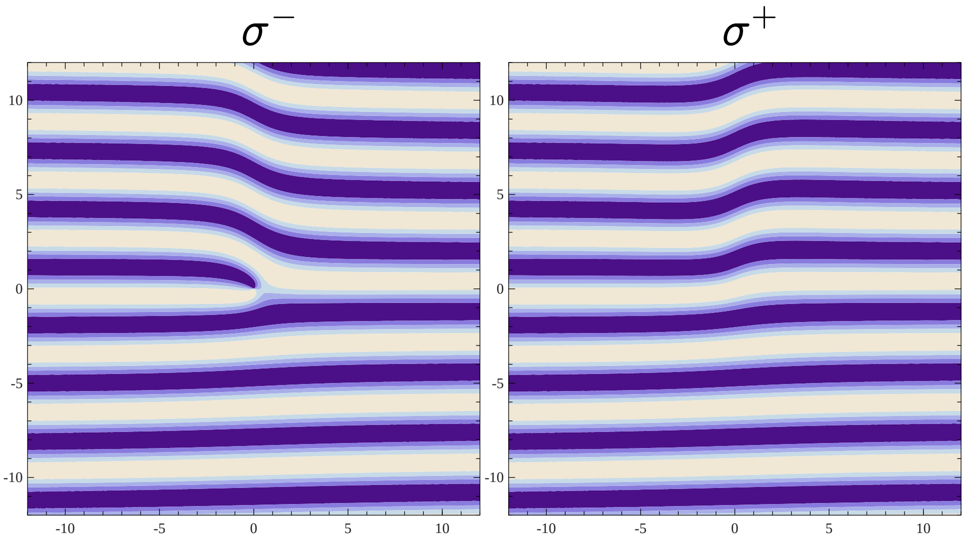
$$\Delta\theta = 0, \qquad \Delta\eta = \epsilon\sin(2\eta).$$

While the phase angle is again $\theta = \pm(1/2)\phi$, the polarization angle η:



In the case of pinning $E^{(p)}(r) \propto r$ for large distances.

String in interference fringes



Effects of TE-TM splitting

The kinetic energy density can be written as

$$T = \frac{\hbar^2}{2}\left\{\frac{1}{m_t}(\nabla_i\psi_j^*)(\nabla_i\psi_j) + \left(\frac{1}{m_l} - \frac{1}{m_t}\right)(\nabla_i\psi_i^*)(\nabla_j\psi_j)\right\}.$$

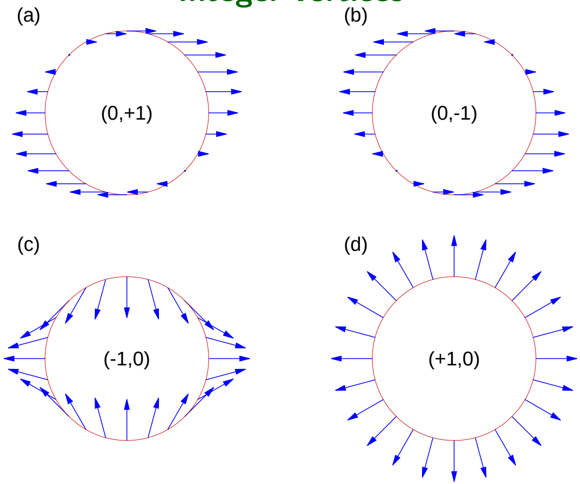
Elastic energy of a single vortex

$$E_{\mathrm{el}}^{(\mathrm{s})} = \frac{\pi\hbar^2 n}{2}\left[\left(\frac{1}{m_l} + \frac{1}{m_t}\right)(k^2 + m^2) + \left(\frac{1}{m_l} - \frac{1}{m_t}\right)(1 - m^2)\delta_{1,k}\right]\ln\left(\frac{R}{a}\right).$$

The main term is the same as without TE-TM splitting with

$$\frac{1}{m^*} = \frac{1}{2}\left(\frac{1}{m_l} + \frac{1}{m_t}\right).$$

Integer vortices



Cases(a − c) : $E_s = \frac{\pi\hbar^2 n}{m^*}\ln(R/a)$, Case(d) : $E_s = \frac{\pi\hbar^2 n}{m_l}\ln(R/a)$.

Since $(1,0) \to (1/2,-1/2)+(1/2,1/2)$, these half-vortices start to interact:

$$V(r_{12}) = \frac{\pi\hbar^2 n}{2}\left(\frac{1}{m_t} - \frac{1}{m_l}\right)\ln\left(\frac{r_{12}}{a}\right).$$

TE-TM splitting "problem"

Considering only kinetic energy terms (in circular basis)

$$i\dot{\psi}_+ = -\frac{1}{2m^*}\left[\Delta\psi_+ + 4\gamma\frac{\partial^2}{\partial z^2}\psi_-\right] + \ldots,$$

$$i\dot{\psi}_- = -\frac{1}{2m^*}\left[\Delta\psi_- + 4\gamma\frac{\partial^2}{\partial z^{*2}}\psi_+\right] + \ldots,$$

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right), \qquad \gamma = \frac{m_t - m_l}{m_t + m_l}.$$

There are ± 2 moment transfer between left and right components.

So, one cannot have solution like $\psi_+ \propto e^{i\phi}$ and $\psi_- \propto \mathrm{const}(\phi)$, because $(\partial^2/\partial z^2)\psi_- \propto e^{-2i\phi}$.

Do half-vortices exist?

Asymptotic behavior

In the elastic region ($r \gg a$):

$$\psi_{\pm}(r \to \infty, \phi) = \sqrt{\frac{n}{2}}\, e^{\mathrm{i}[\theta(\phi) \mp \eta(\phi)]}.$$

$$H_{\mathrm{el}} = \frac{\hbar^2 n}{2m^*} \int d^2 r \left\{ (\nabla\eta)^2 + (\nabla\theta)^2 + 2\gamma \left[\left(\frac{\partial e^{-\mathrm{i}(\theta-\eta)}}{\partial z} \right) \left(\frac{\partial e^{\mathrm{i}(\theta+\eta)}}{\partial z} \right) + \text{c.c.} \right] \right\}.$$

By variation we obtain

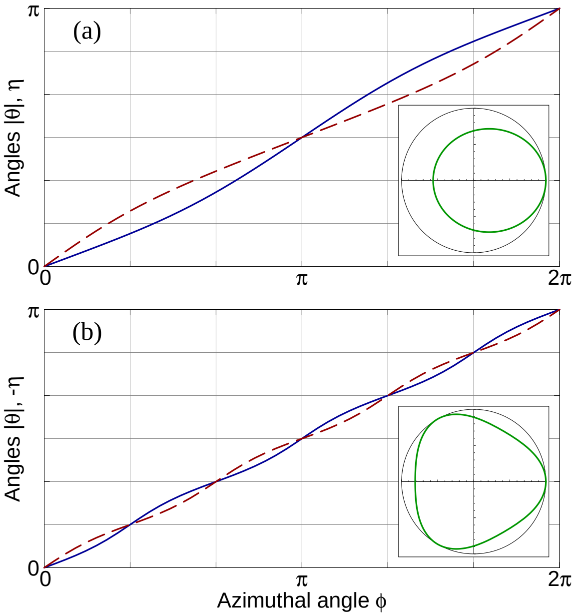
$$[1 - \gamma \cos(2u)]\, \theta'' + 2\gamma \sin(2u) u' \theta' = 0,$$

$$[1 + \gamma \cos(2u)]\, u'' + \gamma \sin(2u) \left(1 - u'^2 - \theta'^2\right) = 0,$$

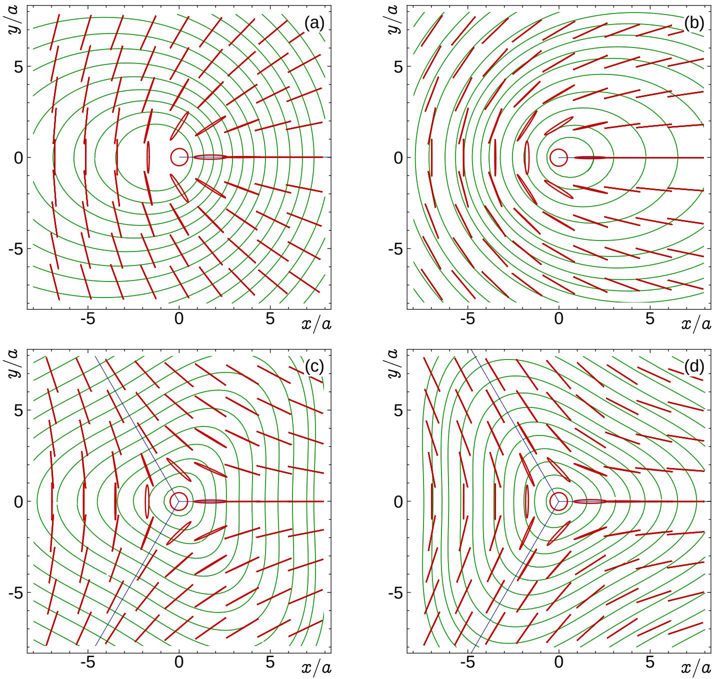
where

$$u(\phi) = \eta(\phi) - \phi.$$

Warping of streamlines

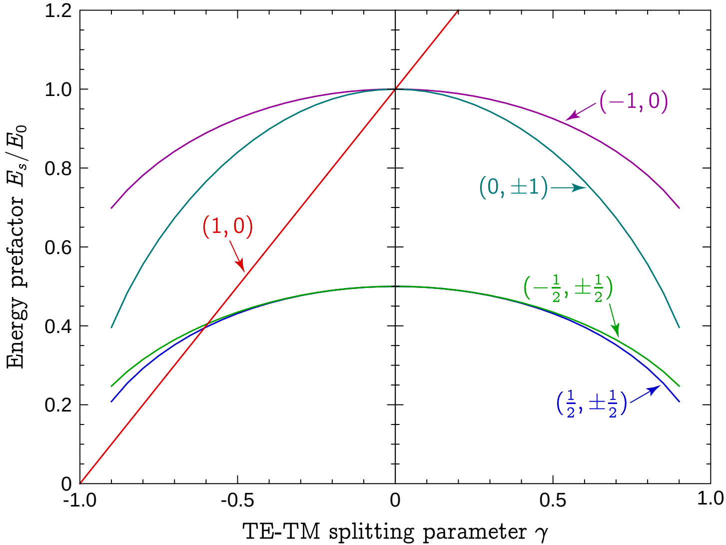


Warped half-vortices (nimerical solutions)



Energies of warped vortices

Vortex energy $E_{\rm vor} = E_c + E_s \ln(R/a)$, and $E_0 = \hbar^2 n/m^*$.



For $m_l \gg m_t$, i.e, for $\gamma \to -1$, the phase transition is defined by proliferation of vortex molecules $(-\frac{1}{2},\frac{1}{2}) - (1,0) - (-\frac{1}{2},-\frac{1}{2})$.

Conclusions

- Vortices in polariton condensates in planar semiconductor microcavities carry two winding numbers (k, m). These numbers can be either integer or half-integer simultaneously. Four half-integer vortices define the BTK transition temperature in the case of unpinned polarization.

- Pinning of polarization results in appearance of strings attached to half-vortices. The TE-TM splitting of polariton bands results in the interaction between left and right half-vortices. Both effects lead to more complicated character of the superfluid phase transition.

- Isolated half-quantum vortices appear in exciton-polariton condensate due to the presence of disorder. They can be detected by simultaneous observation of interference fringes of emitted light in both circular polarizations.

Acknowledgments and References

Collaboration:

- Miller Toledo Solano (CIE-UNAM, México)
- Alexey Kavokin, Tomas Ostatnický (Univ. of Southampton, UK)
- Tim Liew (EPFL, Lausanne, Swiss)
- Konstantinos Lagoudakis, Benoît Deveaud-Plédran (EPFL, Lausanne, Swiss)

References:

Y.G. Rubo, *Phys. Rev. Lett.* 99, 106401 (2007).

T.C.H. Liew, Y.G. Rubo, A.V. Kavokin, *Phys. Rev. Lett.* 101, 187401 (2008).

K.G. Lagoudakis, T. Ostatnický, A.V. Kavokin, Y.G. Rubo, R. André, B. Deveaud-Plédran, *Science* 326(5955), 974 (2009).

M. Toledo Solano, Y.G. Rubo, *J. Phys. Conf. Ser.* 210, 012024 (2010); *Phys. Rev. B* 82, 127301 (2010); *Superlattices and Microstructures* 49, 318 (2011).

Contents